U-duality Between NCOS Theory and Matrix Theory

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Abstract

We show that the NCOS (noncommutative open string) theories on torus T^p ($p \leq 5$) are U-dual to matrix theory on torus with electric flux background. Under U-duality, the number of D-branes and the number of units of electric flux get interchanged. Furthermore under the same U-duality the decoupling limit taken in the NCOS theory maps to the decoupling limit taken in the matrix theory, thus ensure the U-duality between those two class of theories. We consider the energy needed for Higgsing process and some bound states with finite energy and find agreements in both theories.

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1 Introduction

Recently there have been a lot of interests in the NCOS (noncommutative open string) theory [1, 2, 3, 4]. The theory is realized as the world volume theory of D-branes with constant electric field which are taken to the critical value. To extract the worldvolume degrees of freedom only, we need to take some special limit [1, 2] on the background under which all the closed string modes in bulk are decoupled. In contrast to other well-known cases, the higher massive modes of open strings are not decoupled in the limit and thus the theory is not just Yang-Mills theory, but the full open string theory defined on the noncommutative spacetime due to the electric field on the branes. In this way it becomes a consistent theory involving open strings only. In [5] whole class of the NCOS theories, OM (open membrane) theory and ODp (open Dp-brane) theories are defined. See also the independent work on OM theory in [6] and the independent work on OD1 and OD2 theories in [7] with the name (1,1) OBLST and (2,0) OBLST, respectively. All these theories are in the same moduli space if they are defined on torus.

One of the most obvious thing to do toward the understanding of the theory is the study of the duality of the theory, inherited from the 'mother' M/string theory. In the case of (3+1) dimensions the NCOS theory with $F_{01} \neq 0$ naturally arises [2, 3] as the S-dual of the NCYM (noncommutative Yang-Mills) theory with $B_{23} \neq 0$ considered in [8]. (2+1)-dimensional NCOS is S-dual to (2+1)-dimensional U(1) Yang-Mills theory on a M2-brane at low energy [5]. (1+1)-dimensional NCOS theory is S-dual [5, 9, 10] to $(n, 1)$-string theory from which many insightful observations made in [11]. See also [6, 12]. For the related issues and more recent developments in NCOS theories, see [13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

In this paper we study the NCOS theory on torus. In particular we show that there are U dualities between the class of matrix theory [23, 24, 25, 26, 27] defined on torus and the class of the NCOS theory on torus. On one hand it is described by the large N limit of $U(N)$ SYM (super Yang-Mills) theory or their completions, in the sector of M units of electric flux due to the winding fundamental strings.[1] On the other hand it is described by the NCOS theory of M Dp-branes with background N electric flux in the limit of critical

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[1]As is well-known, matrix theory on T^p is described by $U(N)$ SYM (super Yang-Mills) theory on T^p for $p \leq 3$. On higher-dimensional torus, it is true only as the low energy effective description. For complete descriptions, we need extra ingredients [28, 29, 30].

value. This is deeply related to the duality between the theory of N Dp-branes with M fundamental strings and the theory of M Dp-branes with N fundamental strings. This, so-called, N-duality was first discussed in [31] in the context of four dimensional SYM.

However, we would like to stress that we show the same U-duality maps the scaling limit of the backgrounds in one theory to the scaling limit of the backgrounds in the other. On one hand the decoupling limit gives that all the closed string and massive open string modes are decoupled and thus the theory reduces to ordinary Yang-Mills type theory. On the other hand the scaling limit of the background, which are connected by U-duality from the former, results in the decoupling of bulk closed string modes, and gives the theory of open strings only in the background of non-commutative space-time, thus gives NCOS theory. This tells that the dual theory of $U(N)$ SYM with M unit flux of electric field is not $U(M)$ SYM with N electric flux, but rather the full NCOS theory on M D-branes with N unit flux of electric field.

In the case of T^1 and T^2 compactifications, the duality maps strong coupling regime of one theory to the weak coupling regime in the other and vice versa [5]. In the case of T^3 compactification, the Yang-Mills coupling in matrix theory is independent of the open string coupling in the NCOS theory. For higher dimensional torus compactifications, the weak coupling regime of one theory maps to the weak coupling regime of the other. From these dualities, we can easily see that the Higgsing process in matrix theory corresponds to extracting fundamental string in the NCOS theory. This is easily confirmed by computing binding energy which should be identical in both cases.

We also consider some bound states which have finite energy. Once again, under the U-duality, bound states in one theory map to the bound states in the other with the same finite energy, which support the U-duality between those theories. In the case of matrix theory on T^4 and T^5 compactifications, these bound states play essential roles for the complete description of the theory.

In section 2 we shortly review the appropriate scaling limit for each class of theories. In section 3 we explain the U-duality which connects the NCOS theory and the matrix theory for each T^p compactifications. In section 4 we draw some conclusions.

2 Decoupling limit and T^p compactifications

In this section we review the scaling limit taken to get the NCOS theory and the matrix theory, under which all the bulk degrees of freedom are decoupled. We also discuss the torus compactifications of those theories.

2.1 NCOS theory

Consider a Dp-brane with a near critical electric field in the 0,1 direction. The electric field on the brane is given by

$$2\pi\alpha'\epsilon^{01}F_{01} = 1 - \frac{\epsilon}{2} , \tag{1}$$

where critical electric field corresponds to $\epsilon = 0$:

$$\epsilon^{01}F_{01}^{crit} = \frac{1}{2\pi\alpha'} . \tag{2}$$

In the presence of background electric field, or equivalently B_{01} field on the D-brane worldvolume, the metric seen by bulk closed string modes are different from the effective metric seen by the open strings on the D-brane worldvolume. The background metric for closed string is chosen to be [5] [2]

$$g_{\mu\nu} = \eta_{\mu\nu} , \quad g_{ij} = \epsilon\delta_{ij} , \quad g_{MN} = \epsilon\delta_{MN} . \tag{3}$$

The effective string tension of the open string stretched in the x^1 direction is

$$\frac{1}{4\pi\alpha'_e} \equiv \frac{1}{2\pi\alpha'}\left(1 - 2\pi\alpha'\epsilon^{01}F_{01}\right) = \frac{\epsilon}{4\pi\alpha'} . \tag{4}$$

Therefore, while $\alpha' \equiv M_s^{-2}$ sets the scale of closed string modes, $\alpha'_e \equiv M_e^{-2}$ can be considered as the scale of open string modes stretched in the electric direction. The effective open string metric, noncommutativity parameter and string coupling G_o^2 corresponding to (3) can be determined as [32, 33, 8]

$$G_{\mu\nu} = \epsilon\eta_{\mu\nu} , \quad G_{ij} = \epsilon\delta_{ij} , \quad \Theta^{\mu\nu} = 2\pi\alpha'_e\epsilon^{\mu\nu} , \quad G_o^2 = g_s\epsilon^{\frac{1}{2}} . \tag{5}$$

The NCOS limit [1, 2], under which the bulk closed string modes are decoupled, is given by $\epsilon \to 0$ while taking α'_e, G_o fixed. Therefore, under this limit, the scaling is summarized

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[2] $\mu, \nu = 0, 1$ denote the electric directions on the brane and $i, j = 2, 3, \cdots, p$ denote the remaining directions of the brane. $M, N = p + 1, \cdots, 9$ denote the directions transverse to the brane.

as

$$g_{\mu\nu} \sim \mathcal{O}(1) \ , \quad g_{ij} \sim \epsilon \ , \quad g_{MN} \sim \epsilon \ , \quad g_s \sim \epsilon^{-\frac{1}{2}} \ , \quad \alpha' \sim \epsilon \ . \tag{6}$$

Note that if we have compactification along the circle in the non-electric direction of D-brane worldvolume, then the scaling (6) implies the proper radius of the circle behaves as $R_i \sim \epsilon^{\frac{1}{2}} \to 0$. Because of the scaling behavior (6), the scaling of closed string metric, string coupling and string tension after the T-duality is the same as before the T-duality and thus again given by (6). Therefore $(p+1)$-dimensional NCOS theory on S^1 in the non-electric direction on the worldvolume is T-dual to a p-dimensional NCOS theory on dual torus in the transverse direction.

2.2 Matrix theory on T^p

Consider the system with N D0-branes. We want to take the scaling limit [23] under which the low energy description is solely given by the degrees of freedom on N D0-branes. To decouple the bulk closed string modes we take the scaling limit $\alpha' \to 0$ while keeping the energy of stretched open strings between D0-branes with transverse distance r,

$$E = \frac{r}{\alpha'} \ ,$$

fixed. We also keep Yang-Mills coupling constant $g_{YM}^2 = \frac{g_s}{2\pi(\alpha')^{\frac{3}{2}}}$ fixed. These requirements fix the scaling of string coupling and the closed string metric in terms of the scaling of α'. Let

$$\alpha' \sim \epsilon^{\frac{1}{2}} \tag{7}$$

in the scaling limit $\epsilon \to 0$. Then we have

$$g_s \sim \epsilon^{\frac{3}{4}} \ , \quad g_{MN} \sim \epsilon \ , \quad M_p \sim \epsilon^{-\frac{1}{2}} \ , \quad R \sim \epsilon \ , \tag{8}$$

where $M_p = \frac{M_s}{g_s^{\frac{1}{3}}}$ and $R = \frac{g_s}{M_s}$ denote eleven-dimensional Planck scale and eleventh circle radius, respectively. In the $\epsilon \to 0$ limit, we arrive at the theory in which all the bulk closed string modes as well as the massive modes of open strings[3] attached between D0-branes are decoupled. Therefore the remaining relevant degrees of freedom are massless modes of open strings on the worldvolume of D0-branes and the system is described by (0+1)

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[3]Note that, in this setup, the scale of open string modes is the same as the scale of closed string modes, M_s. It is also true in the sector with electric flux.

dimensional SYM quantum mechanics. One may take the discrete light-cone [25] to lift to eleven-dimensions and then by taking the large N limit, we would get the matrix model originally proposed in [24].

One can get similar scaling limit for the system with N Dp-branes ($p \leq 3$) which keeps SYM degrees of freedom only. It is given by

$$\alpha' \sim \epsilon^{\frac{1}{2}} \ , \quad g_s \sim \epsilon^{\frac{3-p}{4}} \ , \quad g_{MN} \sim \epsilon \ , \tag{9}$$

while the metric components in the longitudinal direction are kept fixed in the limit. Therefore, for $p \leq 3$, the system is described by SYM theory with Yang-Mills coupling constant,

$$g_{YM}^2 = (2\pi)^{p-2} g_s (\alpha')^{\frac{p-3}{2}} \ . \tag{10}$$

For $p \geq 4$, the SYM theory is not renormalizable and typically new degrees of freedom enter [28, 29, 30]. In this paper we call all these worldvolume theories of Dp-branes, M2-brane, M5-brane and NS5-branes in the scaling limit (9)[4] as matrix theory.

Now consider the theory defined on p-torus starting from $(0+1)$-dimensional matrix theory set-up [26, 27]. In this case the proper torus radii $R_i \sim \epsilon^{\frac{1}{2}}$ from (8) are vanishingly small, thus it is natural to perform T-duality along the p-torus. After T-duality, N D0-branes turn into N Dp-branes wrapped in the dual torus with radii

$$\Sigma_i = \frac{\alpha'}{R_i} \sim \mathcal{O}(1) \ .$$

The scaling behavior of string coupling and string tension becomes

$$g_s' = g_s \prod_{i=1}^{p} \frac{\sqrt{\alpha'}}{R_i} \sim \epsilon^{\frac{3-p}{4}} \ , \quad \alpha' \sim \epsilon^{\frac{1}{2}} \ , \quad g_{MN} \sim \epsilon \ , \tag{11}$$

which is the same as (9).

3   U-duality of the NCOS theory on T^p

In this section we describe the U-duality transformations of the NCOS theories on T^p. The basic set-up is N D0-branes with M KK momentum along compactified x^1 in the scaling limit (8). As explained in the previous section, after T-duality, it becomes $(1+1)$-dimensional SYM theory. In matrix theory all BPS states in M theory are realized as

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[4]The scaling limit for NS5-brane is the S-dual version of (9).

bound states in SYM or its completions. The physics of M theory in the background of those bound states would be described by the local fluctuations on the corresponding bound states in matrix theory. In the case at hand, the KK momentum turns into fundamental string wrapping S^1. Therefore it becomes the bound state of (N, M) strings which is described by (1+1)-dimensional SYM theory in the sector of M units of electric flux. If we have further torus compactifications, we simply take T-duality on those tori, following the prescriptions given in the last section, and get the matrix theory on T^p with M units of electric flux.

Another way to describe this is to take x^1 - x^{11} flip. This turns out to correspond to the NCOS theory. For T^p with $p = 1, 2$, this reduces to the known results [5], namely, the S-duality between NCOS theory and SYM. For T^p with $p \geq 3$, we encounter new dual relations.

3.1 (1+1)-dimensional NCOS theory

We begin with N sector of matrix theory on S^1 with background M KK momentum along the circle, in the scaling limit (8). The T-duality along the circle, turns the system into the theory of (N,M)-strings which is (1+1)-dimensional $U(N)$ SYM theory with M units flux of electric field. The scaling of the background goes as

$$g_{\mu\nu} = \eta_{\mu\nu} , \quad g_{MN} = \epsilon\delta_{MN} , \quad g_s \sim \epsilon^{\frac{1}{2}}, \quad \alpha' \sim \epsilon^{\frac{1}{2}} . \tag{12}$$

The allowed values of the electric field on the D-string are quantized, and the quantum number can be interpreted as the number of fundamental strings immersed in D-strings. The M unit of electric flux in N D-strings can be expressed as

$$N\frac{2\pi\alpha'\epsilon^{01}F_{01}}{\sqrt{1 - (2\pi\alpha')^2 F^2}} = g_s M . \tag{13}$$

Note that, if we take the scaling in matrix theory, (12), the effective string tension of open strings stretched in x^1 direction is the same as the string tension of closed strings in bulk, $(2\pi\alpha')^{-1}$, as $2\pi\alpha'\epsilon^{01}F_{01} \to 0$, independent of finite N and M. Therefore we can conclude that, in the scaling limit (12), it is described by (1+1)-dimensional ordinary $U(N)$ SYM in the sector of M units of electric flux.

Now we take different path, in which we consider x^1 as eleventh circle. This amounts

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to the x^1 - x^{11} flip. After relabeling the coordinates, the scaling (8) maps to

$$M_p \sim \epsilon^{-\frac{1}{2}} , \quad R \sim \epsilon^{\frac{1}{2}} , \quad R_1 \sim \epsilon , \quad R_M \sim \epsilon^{\frac{1}{2}} . \tag{14}$$

Now after compactification on this new x^{11} circle, the scaling of parameters in ten-dimensional IIA string theory becomes

$$g_s \sim \mathcal{O}(1) , \quad \alpha' \sim \epsilon . \tag{15}$$

After T-duality along x^1, whose radius is vanishingly small, the scaling goes as

$$R_1 \sim \mathcal{O}(1) , \quad R_M \sim \epsilon^{\frac{1}{2}} , \quad g_s \sim \epsilon^{-\frac{1}{2}} , \quad \alpha' \sim \epsilon , \tag{16}$$

which is nothing but the scaling behavior of (1+1)-dimensional NCOS theory (6). Indeed, under these chain of dualities, N D0-branes with M KK momentum become M D-strings with background N units of electric flux, which is given by

$$2\pi\alpha'\epsilon^{01}F_{01} = 1 - \frac{\epsilon}{2} = \frac{g_s\frac{N}{M}}{\sqrt{1 + (g_s\frac{N}{M})^2}} . \tag{17}$$

Therefore ϵ, which represents the deviation of electric flux from critical value, is given by

$$\epsilon = \left(\frac{M}{Ng_s}\right)^2 , \tag{18}$$

from which we find

$$G_o^2 = \frac{M}{N} . \tag{19}$$

Therefore on the one end of the chain of dualities, we have the (1+1)-dimensional NCOS theory of M D-strings with N units of near critical electric flux and on the other end we have $U(N)$ SYM in a sector of M units of electric flux background, as summarized in table 1.

Theory	g_s	α'	x^1 radius	D-string	E-flux
NCOS	$\epsilon^{-\frac{1}{2}}G_o^2$	$\epsilon\alpha'_e$	r_1	M	N
SYM	$\epsilon^{\frac{1}{2}}G_o^{-2}$	$\epsilon^{\frac{1}{2}}G_o^2\alpha'_e$	r_1	N	M

Table 1. 1+1 dimensional theories

The Higgsing energy is identical in both theories [5, 11], as a simple consequence of U-duality. In the SYM theory, the bound state energy of N D-string with M units of electric flux wrapping x^1-circle is given by

$$H_{(M,N)} = \sqrt{\left(\frac{Nr_1}{g_s\alpha'}\right)^2 + \left(\frac{Mr_1}{\alpha'}\right)^2} , \tag{20}$$

in which the first term in the square root divergies much faster than the second one, under the scaling limit (12). The energy of the ground state in $U(N)$ SYM with M units of electric flux above the $U(N)$ SYM ground state with no flux is given by

$$E_{(M,N)} = H_{(M,N)} - H_{(0,N)} = 2\pi r_1 \frac{g_{YM}^2}{2} \frac{M^2}{N} \ , \tag{21}$$

which is finite under the scaling limit (12). Here the gauge coupling of $U(N)$ SYM theory is given by

$$g_{YM}^2 = \frac{g_s}{2\pi\alpha'} = \frac{1}{2\pi G_o^4 \alpha_e'} \ , \tag{22}$$

from the table 1. Therefore, using (19) the ground state energy is given by

$$E_{(M,N)} = 2\pi r_1 \frac{N}{4\pi\alpha_e'} \ , \tag{23}$$

and thus the energy needed to extract D-string is

$$E = 2\pi r_1 \frac{1}{4\pi\alpha_e'} \ , \tag{24}$$

which is the same energy needed to separate an open string with the effective tension $(4\pi\alpha_e')^{-1}$ in (1+1)-dimensional NCOS theory on S^1 with the radius r_1 [5, 11].

One may understand this identification in the following way. Consider the bound state energy of M D-string with N units of electric flux wrapping x^1-circle, which is again given by (20) with interchanged N and M. Now we would like to impose the scaling limit (16) of NCOS theory. Then the second term in the square root divergies much faster than the first term. The ground state energy above the ground state energy of N fundamental strings only is given by

$$E_{(N,M)} = H_{(N,M)} - H_{(N,0)} = 2\pi r_1 N_1 \frac{M^2}{4\pi N^2 g_s^2 \alpha'} = 2\pi r_1 \frac{N}{4\pi\alpha_e'} \ . \tag{25}$$

Therefore we may consider the effective tension of the open string in the NCOS theory as $(4\pi\alpha_e')^{-1}$ which agrees with (4). Of course, this is a simple consequence of U-duality[5]. As will be shown in the following subsections this holds true in the higher-dimensional torus compactifications.

We also note that this dual relation holds even after taking the decompactification limit $r_1 \to \infty$. This can be confirmed by noting that one can also get the same conclusion by simply taking S-duality [5] starting from (1+1)-dimensional NCOS theory.

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[5]The diverging energy, which is subtracted, is also identical in both theories as, again, a consequence of U-duality.

3.2 (2+1)-dimensional NCOS theory

The dual relation between NCOS theory on M D-branes with N electric flux and $U(N)$ SYM with M electric flux holds in higher dimensional T^p compactifications. On T^2 compactifications, we find the dual theory of NCOS theory on M D2-branes with N electric flux is $U(N)$ SYM theory in the sector of M units of electric flux. This is achieved by taking another T-duality starting from the configurations in the previous subsection. It is easily seen by examining the corresponding scaling limit they require. In the matrix theory side, they are given by (11) for $p = 2$, while in the NCOS theory side, after taking T-duality the scaling is given by

$$R_1 \sim \mathcal{O}(1) , \quad R_i \sim \epsilon^{\frac{1}{2}} , \quad R_M \sim \epsilon^{\frac{1}{2}} , \quad g_s \sim \epsilon^{-\frac{1}{2}} , \quad M_s \sim \epsilon^{-\frac{1}{2}} , \tag{26}$$

which is the same scaling as (6). The relation between these two theories are summarized in table 2, in which $g_o^2 \equiv G_o^{-1}(r_2 M_e)^{\frac{3}{2}}$.

Theory	g_s	α'	x^1 radius	x^2 radius	x^{11} radius	M_p^3
NCOS	$\epsilon^{-\frac{1}{2}} G_o^2$	$\epsilon \alpha'_e$	r_1	$\epsilon^{\frac{1}{2}} r_2$	$G_o^2 \alpha_e'^{\frac{1}{2}}$	$\epsilon^{-1} \frac{M_e^3}{G_o^2}$
SYM	$\epsilon^{\frac{1}{4}} g_o^2$	$\epsilon^{\frac{1}{2}} \left(\frac{G_o}{g_o} \right)^{\frac{4}{3}} \alpha'_e$	r_1	$G_o^2 \alpha_e'^{\frac{1}{2}}$	$\epsilon^{\frac{1}{2}} r_2$	$\epsilon^{-1} \frac{M_e^3}{G_o^2}$

Table 2. 2+1 dimensional theories

One can see that these two theories are S-dual each other. The strong coupling regime of NCOS theory, $G_o^2 \to \infty$, maps to the weak coupling limit of SYM theory with gauge coupling,

$$g_{YM}^2 = \frac{g_s}{\sqrt{\alpha'}} = G_o^{-2} \frac{r_2^2}{(\alpha'_e)^{\frac{3}{2}}} . \tag{27}$$

Viewing as T-dual version of (1+1)-dimensional NCOS theory, the (2+1)-dimensional NCOS theory has open string coupling

$$G_o^2 = \frac{M}{N}(r_2 M_e) . \tag{28}$$

In the $U(N)$ SYM theory with M units of electric flux, the ground state energy above the $U(N)$ SYM ground state without electric flux is given by

$$\begin{aligned}
E &= \sqrt{\left(\frac{N r_1 l_2}{2\pi g_s (\alpha')^{\frac{3}{2}}} \right)^2 + \left(\frac{M r_1}{\alpha'} \right)^2} - \frac{N r_1 l_2}{2\pi g_s (\alpha')^{\frac{3}{2}}} \\
&= 2\pi r_1 \frac{g_{YM}^2}{2 l_2} \frac{M^2}{N} ,
\end{aligned} \tag{29}$$

where $l_2 = 2\pi G_o^2 \alpha_e'^{\frac{1}{2}}$ is the proper size of the x^2-circle in SYM theory. Therefore using (27) and (28) the ground state energy is given by

$$E = 2\pi r_1 \frac{N}{4\pi \alpha_e'} \tag{30}$$

Therefore the energy cost needed for taking single D2-brane to infinity is $2\pi r_1 \frac{1}{4\pi \alpha_e'}$, the same energy cost to liberate a fundamental string in (2+1)-dimensional NCOS theory. Again this is a consequence of U-duality.

The above dual relation can be derived by directly interchanging x^2 and x^{11} coordinates in the (2+1)-dimensional NCOS theory on the torus, as can be readily seen in the table 2.

3.3 (3+1)-dimensional NCOS theory

In this section we discuss the case of $p = 3$. Here we have new dual theories of (3+1)-dimensional NCOS theory. If x^3 is also a compact coordinate, we can start from the above (2+1)-dimensional theories, and take T-duality. In this way we find the dual theory of NCOS theory on M D3-branes wrapping on T^3 with N units of electric flux are $U(N)$ SYM theory on T^3 in a sector with M units of electric flux. The (3+1)-dimensional NCOS theory has open string coupling

$$G_o^2 = \frac{M}{N} \frac{r_2 r_3}{\alpha_e'} \ , \tag{31}$$

which is given by T-dualizing (2+1)-dimensional NCOS theory on the torus. The coupling constant of this SYM theory is given by

$$g_{YM}^2 = 2\pi \frac{r_2 r_3}{\alpha_e'} \ , \tag{32}$$

where r_2, r_3 are defined in the table 3. The coupling is independent of the original open string coupling constant G_o! It is not strong-weak coupling duality as in the previous examples and thus one may be able to compare the behavior of one weakly coupled theory to another weakly coupled theory.

Since we are on IIB side, the theory has S-duality. By applying [2] S-duality on the NCOS theory of M D3-branes with N units of electric flux, one obtains NCYM theory of M D3-branes with N units of magnetic flux [8]. The magnetic field is given by [2]

$$F_{23} = \frac{1}{\theta} \quad = \quad \frac{\sqrt{-g}}{g_s} \frac{\epsilon^{01} F_{01}}{\sqrt{1 - (2\pi\alpha')^2 F_{01}^2}}$$

$$= \frac{1}{2\pi G_o^2 \alpha'_e} \, . \tag{33}$$

Thus the dimensionless parameter $\Theta = \frac{1}{2\pi r_2 r_3}\theta$ defined in [8] has rational value $\Theta = \frac{M}{N}$ after using (31). In this case it was shown in [8] that there is a T-duality which maps NCYM theory to ordinary SYM theory.

By applying S-duality on the ordinary $U(N)$ SYM theory in the sector of M units of electric flux, one gets ordinary $U(N)$ SYM theory in the sector of M units of magnetic flux with gauge coupling

$$(g'_{YM})^2 = \frac{4\pi^2}{g_{YM}^2} \, . \tag{34}$$

In fact this $U(N)$ SYM theory with M units of magnetic flux is the one mentioned above as a T-dual theory to the NCYM theory.

All these are summarized in the table 3 where $g_o^2 \equiv \frac{g_{YM}^2}{2\pi} = \frac{r_2 r_3}{\alpha'_e}$.

Theory	g_s	α'	x^1 radius	x^i radius	g_{MN}
NCOS	$\epsilon^{-\frac{1}{2}} G_o^2$	$\epsilon \alpha'_e$	r_1	$\epsilon^{\frac{1}{2}} r_i$	$\epsilon \delta_{MN}$
NCYM	$\epsilon^{\frac{1}{2}} G_o^{-2}$	$\epsilon^{\frac{1}{2}} G_o^2 \alpha'_e$	r_1	$\epsilon^{\frac{1}{2}} r_i$	$\epsilon \delta_{MN}$
SYM with E-flux	g_o^2	$\epsilon^{\frac{1}{2}} \left(\frac{G_o}{g_o}\right)^2 \alpha'_e$	r_1	$\left(\frac{G_o}{g_o}\right)^2 r_i$	$\epsilon \delta_{MN}$
SYM with M-flux	g_o^{-2}	$\epsilon^{\frac{1}{2}} G_o^2 \alpha'_e$	r_1	$\left(\frac{G_o}{g_o}\right)^2 r_i$	$\epsilon \delta_{MN}$

Table 3. 3+1 dimensional theories

Here again, the Higgsing energy in SYM theory corresponds to the energy required to extract a string in NCOS theory. Since the case with electric flux was presented in the previous sections for lower dimensional D-branes, here we present only the case with magnetic flux. In the $U(N)$ SYM theory with M units of magnetic flux, the ground state energy above the $U(N)$ SYM ground state without magnetic flux is given by

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\begin{aligned}
E &= \sqrt{\left(\frac{N r_1 l_2 l_3}{g_s (2\pi \alpha')^2}\right)^2 + \left(\frac{M r_1}{g_s \alpha'}\right)^2} - \frac{N r_1 l_2 l_3}{g_s (2\pi \alpha')^2} \\
&= 2\pi r_1 \frac{2\pi^2}{(g'_{YM})^2 l_2 l_3} \frac{M^2}{N} \, ,
\end{aligned} \tag{35}
$$

where $l_i = 2\pi \frac{G_o^2}{g_o} r_i$ is the proper size of the x^i-circle in SYM theory. Therefore using (31), (32) and (34), the ground state energy is given by

$$E = 2\pi r_1 \frac{N}{4\pi \alpha'_e} \, , \tag{36}$$

And therefore the Higgsing energy in $U(N)$ SYM with M units of magnetic flux coincides with the energy cost to send a wrapping string along x^1-direction to infinity.

3.4 OM theory

Next we consider the T^4 compactification. In the matrix theory side, after taking T-duality along the torus, the string coupling diverges, as can be seen from (11) with $p = 4$. Therefore we are considering the strong coupling limit of IIA string theory, which is an eleven dimensional M theory. D4-branes become M5-branes wrapping the eleventh dimension whose radius is now finite and is given by

$$R_{11} = g_s \sqrt{\alpha'} \sim \mathcal{O}(1). \tag{37}$$

The low energy (4+1)-dimensional SYM theory on D4-brane worldvolume is not renormalizable and breaks down at energies of order $E \sim \frac{1}{g_{YM}^2}$, where new degrees of freedom should be taken into account. Since the SYM coupling is given by $g_{YM}^2 = (2\pi)^2 R_{11}$, the new degrees of freedom are the KK momentum in eleventh direction, i.e. D0 particles in ten dimensional point of view.

The eleven dimensional Planck scale becomes

$$M_P = \frac{M_s}{(g_s)^{1/3}} \sim \epsilon^{-1/6} \longrightarrow \infty \; . \tag{38}$$

Therefore the bulk degrees of freedom are decoupled and it becomes the theory of N M5 branes in eleven dimensions, i.e. (0,2) theory [28, 29], with M unit flux of background worldvolume three form field strength.

In the dual NCOS theory, after taking T-duality along T^4, the string coupling diverges, $g_s \sim \epsilon^{-\frac{1}{2}}$, so we may lift to the eleven dimensions again. Here the radius of eleventh direction is fixed under the limit and in fact is the same as the eleventh radius in the matrix theory above. The eleven dimensional Planck scale becomes $M_P \sim \epsilon^{-1/3} \rightarrow \infty$. Therefore (4+1)-dimensional NCOS theory can be considered as the compactification of (5+1)-dimensional OM theory[5] with finite radius. Indeed it gives the right scaling for the OM theory with a background worldvolume three form field strength

$$H_{012} = \frac{2\pi F_{01}}{R_{11}} = \epsilon^{-1} 2M_{\text{eff}}^3 \left(1 - \frac{\epsilon}{2}\right) \; , \tag{39}$$

where $R_{11} = G_o^2 (\alpha_e')^{\frac{1}{2}}$ is the radius of the eleventh coordinate. $\frac{M_{\text{eff}}^3}{4\pi^2} \equiv \frac{M_e^3}{8\pi^2 G_o^2}$ is the effective tension of a open membrane stretched in the x^1 and x^{11} directions [5]. This can be

confirmed in the same way as the case of effective string tension in section 3.1. One can consider the bound state energy of D4-branes with electric flux in the NCOS scaling limit (6) and then reexpress in the eleven dimensional variables. The relation between the (4+1)-dimensional NCOS theory on M D4-branes and matrix theory with M units of electric flux are summarized in table 4 with $g_o^2 \equiv G_o(r_2 r_3 r_4 M_e^3)^{\frac{1}{2}}$.

Theory	g_s	α'	x^i radius	x^{11} radius	M_p^3
NCOS	$\epsilon^{-\frac{1}{2}} G_o^2$	$\epsilon \alpha'_e$	$\epsilon^{\frac{1}{2}} r_i$	$G_o^2 \alpha_e'^{\frac{1}{2}}$	$\epsilon^{-1} \frac{M_e^3}{G_o^2}$
matrix theory with E-flux	$\epsilon^{-\frac{1}{4}} g_o^2$	$\epsilon^{\frac{1}{2}} \left(\frac{G_o}{g_o}\right)^4 \alpha'_e$	$\left(\frac{G_o}{g_o}\right)^4 r_i$	$G_o^2 \alpha_e'^{\frac{1}{2}}$	$\epsilon^{-\frac{1}{2}} \left(\frac{g_o}{G_o}\right)^4 \frac{M_e^3}{G_o^2}$

Table 4. 4+1 dimensional theories

The NCOS theory on torus has other degrees of freedom with finite energy. These are D-branes wrapping on the non-electric directions. In the (4+1)-dimensional NCOS theory, D0-branes and D2-branes wrapping on 2-torus can have finite energy. Especially D0-branes have the energy

$$E = \frac{1}{R_{11}} = \frac{M_e}{G_o^2}$$

thus in the strong coupling regime, $G_o > 1$, D0-brane excitation has lower energy than the string stretched in the x^1 direction. Under U-duality we are considering, the D0-brane in NCOS theory maps to D0-brane in matrix theory on T^4. This identification can be easily established as both theories have the same X^{11} radius and thus D0-branes have the same excitation energies in both theories.

3.5 Near critical NS5-brane theory

Finally we consider the compactification on T^5. In the matrix theory side, after taking T-duality along T^5, (11) tells us that it becomes the strong coupling limit of D5-branes in type IIB theory. Therefore we perform the S-duality and then it becomes the theory of N NS5-branes in type IIB theory with M unit flux background. Note that the unit flux of electric field on the worldvolume of D5-branes can be traded for the unit B_{01} NS 2-form field by the gauge transformation, which turns into the unit C_{01} RR gauge field under S-duality. The theory has vanishingly small string coupling, $g_s \sim \epsilon^{\frac{1}{2}}$, and finite closed string tension in the limit $\epsilon \to 0$. The original gauge coupling (10) turns, under S-duality, into $g_{YM}^2 = (2\pi)^3 \alpha'$. The low energy effective gauge theory on the NS5-branes is not

renormalizable and breaks down at energies of order $E \sim \frac{1}{g_{YM}} \sim \frac{1}{\sqrt{\alpha'}}$ where new degrees of freedom enter. They are strings with tension $(2\pi\alpha')^{-1}$ and thus can be interpreted as the fundamental strings on NS5-branes. Since $g_s \to 0$, the bulk degrees of freedom are decoupled and those strings remain on the NS5-branes. This theory is known as LST (little string theory) [30]. By taking T-duality, one can get LST on type IIA NS5-branes.

In the NCOS side, after performing T-duality along T^5, the theory becomes (5+1)-dimensional NCOS theory on D5-brane, where the open fundamental string stretched in the electric direction remains light degrees of freedom. In this case the theory is in type IIB side and thus we can take S-duality. The resultant theory is the decoupled theory on the worldvolume of the NS5-brane with a near critical background C_{01} RR gauge field. In this case the light degrees of freedom are open D-strings due to a near critical two form RR potential and the theory is called OD1 theory [5] or (1,1) OBLST [7]. These are S-dual version of fundamental strings with a near critical two form NS-NS potential, or electric field on the original D5-brane. When the OD1 theory is defined on the torus, as in the present case, we can take T-duality and the theory becomes ODp theory on the NS5-brane worldvolume, whose light degrees of freedom are open Dp-branes stretched in the direction of RR C_{p+1} fields. Note that for even p the theory is the worldvolume theory of type IIA NS5-brane and for odd p it is the worldvolume theory of type IIB NS5-brane, just like in the matrix theory. In table 5, the relations between these theories are summarized. Here we show the case of OD1 theory only. Other ODp theories for $p \neq 1$ can be easily given by taking appropriate T-duality.

Theory	g_s	α'	x^i radius
NCOS theory	$\epsilon^{-\frac{1}{2}}G_o^2$	$\epsilon\alpha'_e$	$\epsilon^{\frac{1}{2}}r_i$
OD1 theory	$\epsilon^{\frac{1}{2}}G_o^{-2}$	$\epsilon^{\frac{1}{2}}G_o^2\alpha'_e$	$\epsilon^{\frac{1}{2}}r_i$
LST with E-flux	$\epsilon^{\frac{1}{2}}G_o^{-2}$	$G_o^4\frac{\alpha_e'^3}{r_2r_3r_4r_5}$	$G_o^2\frac{\alpha_e'^2}{r_2r_3r_4r_5}r_i$

Table 5. 5+1 dimensional theories

The (5+1)-dimensional NCOS theory and OD1 theory on the torus also has additional degrees of freedom with finite energy. These are related to the little strings in LST under U-duality. In the NCOS theory they are D-strings wrapped in one toroidal direction. In the OD1 theory they are fundamental strings wrapped in the same direction [6]. By

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[6]After T-duality the theory becomes the ODp theory on the dual torus and these finite energy excitations correspond to KK momentum in the T-dualized direction.

U-dual chain, all these map to the little string wrapped in the same direction in LST. For example, one can easily see from the table 5 that the energy of fundamental strings wrapped in the x^5-coordinate in OD1 theory is given by

$$E = \frac{r_5}{G_o^2 \alpha_e'} \ ,$$

which is the same as the energy for the little string in LST and D-string in NCOS theory, wrapped in the same direction. One can also easily confirm that, along the same line of arguments shown in the lower dimensional toroidal compactifications, all these theories on torus have the same Higgsing energy due to U-duality. The corresponding process in NCOS theory is the separation of open fundamental string wrapped in the x^1 direction to the infinity. In ODp theory with $C_{0\cdots p} \neq 0$ it corresponds to the separation of Dp-brane wrapped in the $x^1, \cdots x^p$ directions to the infinity. Note that in this case U-duality maps weakly coupled regime of OD1 theory to weakly coupled regime of LST.

4 Discussions

Now there are two (or more) different ways to study the worldvolume theory on N Dp-branes with M units of electric flux and their analogues in M5-branes and NS5-branes. One is to take the scaling limit (11), under which all the bulk degrees of freedom and massive modes of open strings attached to the branes are decoupled and the theory reduces to $U(N)$ SYM theory or its generalization for $p \geq 4$ in the sector of M units of electric flux. Another way to study the system is to take the scaling limit (6), under which the bulk closed string modes are again decoupled, while the massive modes of open strings stretched in the electric direction can not be neglected and thus the theory becomes full open string theory defined on noncommutative spacetime. What has been shown in this paper is that these are connected by U-duality inherited from M theory, with the exchange of the number of branes and the number of units of electric flux, i.e. they lie in the same moduli space. We can identify the bound states with finite energy in the NCOS theory with the states in the matrix theory. Some of these states correspond to the elementary excitations in matrix theory on T^4 and T^5 compactifications. These dual relations nicely explain that NCOS theories reduces to matrix theory at low energies [5].

From the dual relations between NCOS theory and DLCQ M theory, one would get lots of useful information and insights on those theories. In particular, it is quite striking

that we can study the large N limit of matrix theory on torus, in the sector of M unit of electric flux, using the NCOS theory.

This U-duality would have deep implications in the dual gravity descriptions of those theories. We will return this issue in the near future.

Acknowledgements

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